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Gerald Beresford Whitham 13th December, 1927 --- 26th January, 2014

Citation for published version:

Minzoni, AA & Smyth, N 2015, 'Gerald Beresford Whitham 13th December, 1927 --- 26th January, 2014', *Biographical Memoirs of Fellows of the Royal Society*, vol. 61, pp. 555-577.
<https://doi.org/10.1098/rsbm.2014.0026>

Digital Object Identifier (DOI):

[10.1098/rsbm.2014.0026](https://doi.org/10.1098/rsbm.2014.0026)

Link:

[Link to publication record in Edinburgh Research Explorer](#)

Document Version:

Peer reviewed version

Published In:

Biographical Memoirs of Fellows of the Royal Society

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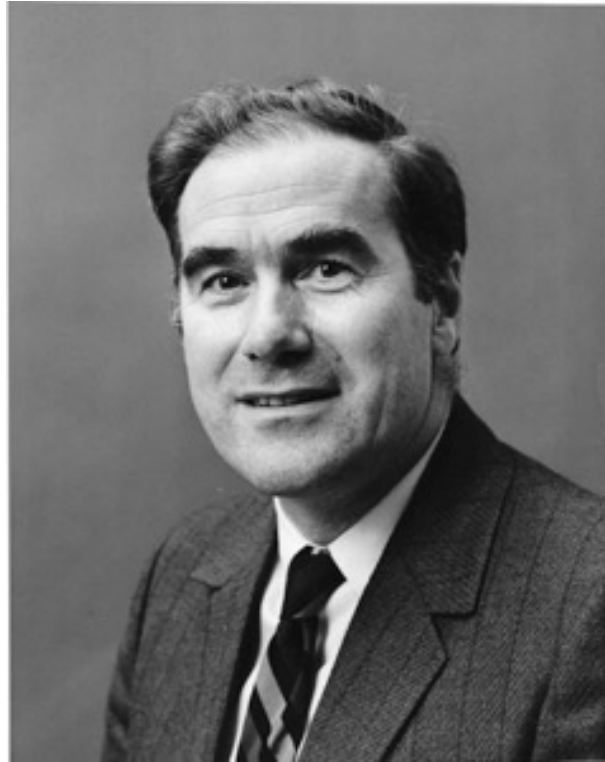
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GERALD BERESFORD WHITHAM
13th December, 1927 — 26th January, 2014



GERALD BERESFORD WHITHAM

13th December, 1927 — 26th January, 2014

Elected FRS 18th March, 1965

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Abstract

Professor G.B. Whitham, FRS, was one of the leading applied mathematicians of the twentieth century. His original, deep and insightful research into nonlinear wave propagation formed the foundation of and mathematical techniques for much of the current research in this area. Indeed, many of these ideas and techniques have spread beyond wave propagation research into other areas, such as reaction-diffusion, and has influenced research in pure mathematics. His textbook *Linear and Nonlinear Waves*, published in 1974, is still the standard reference for the mathematics of wave motion. G.B. Whitham was also instrumental in building from scratch the Department of Applied Mathematics at the California Institute of Technology and, through choosing key people in new, promising research areas, making it into one of the leading centres of applied mathematics in the world, with an influence far beyond its small size. During his academic career, G.B. Whitham received major awards and prizes for his research. He was elected a Fellow of the Royal Society in 1965, a Fellow of the American Academy of Arts and Sciences in 1959 and was awarded the Norbert Wiener Prize for Applied Mathematics in 1980.

Gerald Beresford Whitham was born in Halifax, West Yorkshire on the 13th December, 1927 and remained a proud Yorkshireman all his life. His family was poor. His father, Harry Whitham, also born in Halifax, was a Great War veteran who found difficulty finding work after the war, eventually joining the Royal Air Force. His mother, Elizabeth Ellen Whitham (née Howarth), born in Elland, Yorkshire, worked long hours in local woollen mills to help support the family. Whitham had an older brother, Denis Talbot Whitham, born on the 16th September, 1923 who served in the Royal Air Force in a VIP squadron during the Second World War and became a pilot for B.O.A.C. and then British Airways after the war. Whitham's mother had obtained a scholarship to go to grammar school when she was a girl, but had to leave school at primary level to work to bring in money for her family. She regretted this the rest of her life and made sure that her sons had the best educational opportunities possible. Whitham received his secondary education at Elland Grammar School, Elland, Yorkshire, England from 1938 to 1945, with funding from a Country Minor Scholarship from the West Riding of Yorkshire. The headmaster of Elland Grammar recognised Whitham's mathematical talent and gave him private lessons. Whitham entered the University of Manchester in 1945, funded by a County Major Scholarship from the West Riding of Yorkshire and a Brooksbank Exhibition scholarship from Elland Council. Whitham chose the University of Manchester as "local folklore" said that it was a very good university. During his undergraduate studies, Whitham won the Dalton Mathematics Prize in 1946 and 1947. He received a first class B.Sc. honours degree in Mathematics in 1948. Whitham later said that both Sydney Goldstein, FRS, the Beyer Chair of Applied Mathematics at the

university, who was a notable leader in fluid mechanics research, and M. James Lighthill, later Sir James Lighthill, FRS, were big influences on him and his research while he was at Manchester. The Department of Mathematics at the University of Manchester was then a leading centre of applied mathematics research in fluid mechanics. Also in the Department of Mathematics at the University of Manchester at this time was the famous number theorist Kurt Mahler, one of the many scientific exiles from Hitler's Germany. Whitham used to tell the story dating from when he was an undergraduate student at the University. Students had to sign in at lectures by writing down their names on a sheet of paper. This was not taken seriously by the students, with Karl Gauss and other famous mathematicians apparently at the lectures, much to the consternation of Mahler, who was much too serious about mathematics to appreciate the humour.

Whitham then moved to post-graduate study, for which he was funded by a D.S.I.R. (Department of Scientific and Industrial Research) grant for research, starting with an M.Sc. degree. Whitham had approached Lighthill about doing an M.Sc. and Ph.D. on general relativity. Lighthill told him that he did not work on general relativity, but that he did work on fluid dynamics. This encounter started Whitham's long and distinguished career in wave theory. As an aside, after Whitham retired, he intended to start research on general relativity, but as far as the authors are aware, this did not lead to any new work. The context of this interest in shock waves is that this was the period immediately after the Second World War when there was intense interest in supersonic flight and the behaviour of shock waves generated in supersonic fluid flow. The jet engine had been invented by Sir Frank Whittle in the United Kingdom and independently developed in that country and Germany during the war, leading to the first jet aircraft. The jet engine enabled the limitations of propeller driven flight to be overcome, enabling supersonic flight. Lighthill later remarked in relation to Whitham that

"I feel some pride at having, nineteen years ago, chosen as the problem to give to the first really good research student I had, the estimation of pressure pulses transmitted to great distances by bodies moving at supersonic speed" (Lighthill, 1967).

This led to the award of an M.Sc. thesis on "The behaviour of supersonic flow past a body of revolution" in 1949 and a Ph.D. degree for the thesis "Propagation of a spherical blast" in 1953, both at the University of Manchester. The work for the Ph.D. was completed in 1951, but the degree was awarded in 1953 due to Whitham being in the United States at the Courant Institute of Mathematical Sciences in the intervening period. Whitham met his wife, Nancy Whitham (née Lord), while he was a Ph.D. student at Manchester. Nancy was an undergraduate Mathematics student and they first met when Whitham was her tutor for a mathematics course. Nancy Lord was born on the 29th December, 1929 in Oldham, Lancashire, the daughter of Frank Lord and Amy Lord (née Buckley). G.B. Whitham and Nancy Lord married on the 1st September, 1951 in Manchester. They had three children, Ruth, Michael and Susan.

Academic Career

G.B. Whitham's academic career spanned appointments in two countries, the United Kingdom and the United States of America, with visiting appointments in Australia, the United Kingdom and India. In 1950, Richard Courant visited Manchester University. Richard Courant founded the Institute of Mechanics and Applied Mathematics, now the Courant Institute



Figure 1: Gerald B. Whitham and Nancy Whitham on their wedding day, 1st September, 1951 in Manchester, England.

of Mathematical Sciences, at New York University. The Courant Institute attracted many leading mathematicians forced out of Europe in the 1930's, including Kurt Otto Friedrichs and Fritz John. The needs of victory in the Second World War turned many of these pure mathematicians to applied mathematics research. This included major work on shock waves in compressible fluids, which was later published in book form as *Supersonic Flow and Shock Waves* (Courant and Friedrichs, 1976, originally published in 1948). The Courant Institute was already a leading centre of mathematics research, established by Richard Courant, a student of David Hilbert, in 1936. This visit in 1950 by Richard Courant resulted in Whitham being invited to the Institute of Mechanics and Applied Mathematics as a research associate from 1951 to 1953. Whitham felt very welcome there with Courant giving Nancy a job and making sure that they had enough money. In the European tradition, Courant, Friedrichs, Fritz John and J.J. Stoker showed the Whithams “tremendous kindness” with family get togethers. While the leading people at the Institute had an appreciation for applied mathematics, the majority were traditional pure mathematicians and Whitham's first seminar “shocked them” with its intuitive solution of the equations for sonic booms. Whitham also visited Princeton and during one visit attended a seminar by John von Neumann at which he politely told him that he had “missed a point” in fluid dynamics, which stunned both the audience and von Neumann himself.

Whitham was asked to stay at the Courant Institute, but after reflection he and Nancy felt that they wanted to return to the United Kingdom. Whitham also felt that the emphasis at Courant was too pure for his tastes. So Whitham moved back to Manchester University as a lecturer in applied mathematics, a position he held from 1953 to 1956. However, he then returned to the Courant Institute as an associate professor in applied mathematics in 1956 as he was missing the atmosphere there, remaining until 1959. He also felt that the United States offered a much higher standard of living, which was marked in those post-war years. Again, he felt that Courant lacked people interested in applied mathematics based on physical reasoning, with too much emphasis on theorems and his previous restlessness returned. Whitham was offered a full professorship at Courant, but moved in 1959 to the Massachusetts Institute of Technology (MIT) as a professor of mathematics. This move was helped by George Carrier and Sydney Goldstein at Harvard University and C.C. Lin at MIT, Goldstein having moved to Harvard via the Technion in Israel. The professorial appointment

at MIT takes a subsequent light given the traditional good humoured rivalry between Caltech and MIT. Whitham remained on faculty at MIT until 1962.

Whitham found MIT to be a very large university after his experience at New York University, which he felt to be small and friendly. Whitham was thinking of returning to Courant when, in the summer of 1960, Hans Liepmann of GALCIT, Graduate Aeronautical Laboratories of the California Institute of Technology, later its Director, visited Harvard. GALCIT had been founded by Theodore von Karman and was the leading centre for fluid mechanics and aeronautics, with many famous scientists and engineers at the time, such as Clark Millikan, H. Liepmann, L. Lees, P.A. Lagerstrom, J.D. Cole, D.E. Coles, F.E. Marble and A. Roshko. He suggested that Whitham visit GALCIT, with a possibility of this leading to an appointment. Whitham duly came to the California Institute of Technology (Caltech) as a visiting Professor of Applied Mechanics from 1961 to 1962. Whitham enjoyed Caltech immensely as it had a similar style and small size as Courant, but had a large number of people with similar interests in physical applied mathematics and fluid mechanics. His previous negative image of California as “Tinseltown” and “swimming pools” quickly disappeared.

The major legacy of Whitham’s scientific vision and leadership is the Department of Applied Mathematics at the California Institute of Technology, now the Applied and Computational Mathematics option within the Department of Computing and Mathematical Sciences. The Department of Applied Mathematics at the California Institute of Technology became so much associated with Whitham that it was known in applied mathematics circles as “Whitham’s department.” Whitham moved to Caltech permanently in 1962 and was a professor of aeronautics and mathematics from 1962 to 1967. At the time, there was an influential group of applied mathematicians in the Department of Aeronautics, now the Department of Aerospace, working on asymptotic methods as applied to fluid mechanics problems, including Paco A. Lagerstrom, Julian D. Cole and Saul Kaplun, but no coherent group working on applied mathematics in a broad sense. In 1967 Whitham was instrumental in setting up a separate Department of Applied Mathematics down the corridor from the Department of Aeronautics. He served as chairman of the Committee on Applied Mathematics from 1962 to 1971 and then the executive officer (head) of department from 1971 to 1980. This department included Lagerstrom and Cole, Kaplun having passed away in 1964. He recruited an outstanding group of applied mathematicians working in a wide variety of areas for this new department. These include Philip G. Saffman, FRS in 1964, Donald S. Cohen in 1965, Herb B. Keller in 1967, Heinz-Otto Kreiss in 1978 and Bengt Fornberg in 1974. This formed a small, but highly influential group which became one of the leading centres of applied mathematics in the world. The later appointment of Fornberg, Keller and Kreiss shows an early recognition of the importance which numerical analysis was to play in applied mathematics. Whitham was a professor of applied mathematics at the California Institute of Technology from 1967 to 1983 and the Charles Lee Powell Professor from 1983 to his retirement in 1998.

In the process of building the Department of Applied Mathematics at the California Institute of Technology, Whitham combined two ingredients. The first was that Whitham, through his work on shock waves, realised that the understanding of nonlinear problems and nonlinear equations needed new ideas. At the time he started the department, bifurcation theory and nonlinear analysis were just starting to contribute to the study of nonlinear problems, with potential applications in a diverse range of fields, including fluid mechanics, chemical reactions and oscillator problems. Whitham realised the importance of these new fields, not directly through his own research, but by building a group of applied mathematicians who knew these new areas and who were also knowledgeable about the underlying science of po-

tential application fields and so could successfully work on applications with other scientists and engineers. The work of Cohen, Keller and Saffman on bifurcation theory lead to both new mathematical ideas and new applications in a diverse range of scientific and engineering fields. Moreover, the now widely used and standard numerical idea of continuation originated through this group at this time. The second ingredient was the realisation that numerical solutions of equations could help to stimulate new understanding which went beyond the actual accurate solution itself. One result of this insight was the now classic paper on nonlinear dispersive phenomena [16], which numerically studied a range of important problems in the area, particularly those for solitary wave equations, and which gave the first proper study of undular bores. In general, the work of Fornberg and Kreiss on hyperbolic and dispersive waves originated much of the current understanding of the numerical solution of multiscale problems and coherent structures. As is well known, there is no Nobel Prize in Mathematics, the reasons for which are the subject of many humorous tales. However, Whitham did like to say that the Department of Applied Mathematics at Caltech did have one Nobel Prize, that of Robert C. Merton, who won the Nobel Prize in Economics in 1997. Merton obtained an M.Sc. in Applied Mathematics from Caltech in 1967, but switched to a Ph.D. in Economics at MIT. In his role as head of department, Whitham helped Merton make the transition from applied mathematics to economics.

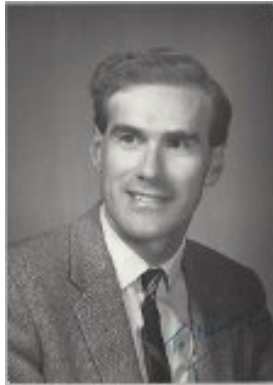


Figure 2: G.B. Whitham as a newly arrived Professor of Mathematics at the Massachusetts Institute of Technology in 1959.

Whitham enjoyed travelling and had visiting appointments at a number of universities. He was highly valued in this role, both for his stature in applied mathematics and for his well deserved reputation as a lecturer who could give outstanding courses on a wide variety of subjects, including his speciality of nonlinear waves. Whitham had a particular regard for Australia, which was shared with his wife, Nancy. He was a visiting professor at the Department of Mathematics at the University of Queensland from July to October, 1980, where, incidentally, one of the authors (NFS) first met him as an undergraduate student. Whitham also knew Professor Ted Buchwald of the Department of Applied Mathematics at the University of New South Wales, located in Sydney, from his days at the University of Manchester. Professor Buchwald arranged for him to be a visiting professor at the university from July to August, 1984 and January to April, 1989. Whitham returned to the United Kingdom as a visiting scholar at the University of Oxford from April to June, 1981. Whitham also visited the Tata Institute of Fundamental Research at the Indian Institute of Science, Bangalore, India from January to March, 1978 and November to December, 1980. As part of

this visit, he delivered a course on wave motion, which was published as *Lectures on Wave Propagation* [17]. During his time in both Australia and India, Whitham and his family travelled extensively around both countries. Whitham documented all the family travels with a vast collection of colour slides, spanning 1960 to 1998. He would often say while photographing some landmark, “Stand over there for scale.” He meticulously catalogued and described all the slides in a journal— this has become a family treasure.

An interesting story relates to one of Professor and Mrs. Nancy Whitham’s visits to Australia when they took a driving holiday around Tasmania. They were driving in a wilderness area on the western side of Tasmania when they saw an animal they did not recognise cross the road. At the next township they asked a woman what it was. She said that they should not have seen such an animal as it was a Tasmanian tiger (or Tasmanian wolf) and they are extinct! There have been many alleged sightings of the Tasmanian wolf. What distinguishes this one is that neither Professor nor Mrs. Whitham knew such an animal existed before they saw it in Tasmania that day.

Scientific Honours

G.B. Whitham’s outstanding research in and influence on applied mathematics was recognised with awards from a number of bodies in both the United Kingdom and the United States of America. The peak honour was being elected a Fellow of the Royal Society on the 18th March, 1965. Whitham was elected a Fellow of the American Academy of Arts and Sciences in 1959, just three years after he arrived in the United States. In 1980 Whitham was awarded the Wiener Prize in Applied Mathematics, one of the major international awards in applied mathematics. The description of the Wiener Prize states that

“The Wiener Prize is awarded for an outstanding contribution to “applied mathematics in the highest and broadest sense.” Awarded jointly by the American Mathematical Society and the Society for Industrial and Applied Mathematics. This prize was established in 1967 in honour of Professor Norbert Wiener and was endowed by a fund from the Department of Mathematics of the Massachusetts Institute of Technology.”

Finally, Whitham was on the Oceanography Panel of the President’s Scientific Advisory Council from 1965 to 1966.

Areas of Research

The applied mathematics research of G.B. Whitham in wave motion falls into the two general areas of waves governed by hyperbolic equations and waves governed by dispersive equations. This is also the general division of material in his classic text *Linear and Nonlinear Waves* [15], itself based on his long running course at the California Institute of Technology, AMa152, “Linear and Nonlinear Waves.” *Linear and Nonlinear Waves* is still the standard reference in the area and has not been surpassed in the forty years since its publication. Reading and re-reading sections of the book is still a rewarding task which yields new insights. In general, the first part of Whitham’s career up until the 1960’s was devoted to hyperbolic waves, initiated by his study of shock waves in supersonic flow, with the latter part of his career devoted to dispersive waves, as exemplified by water waves.

Hyperbolic Waves

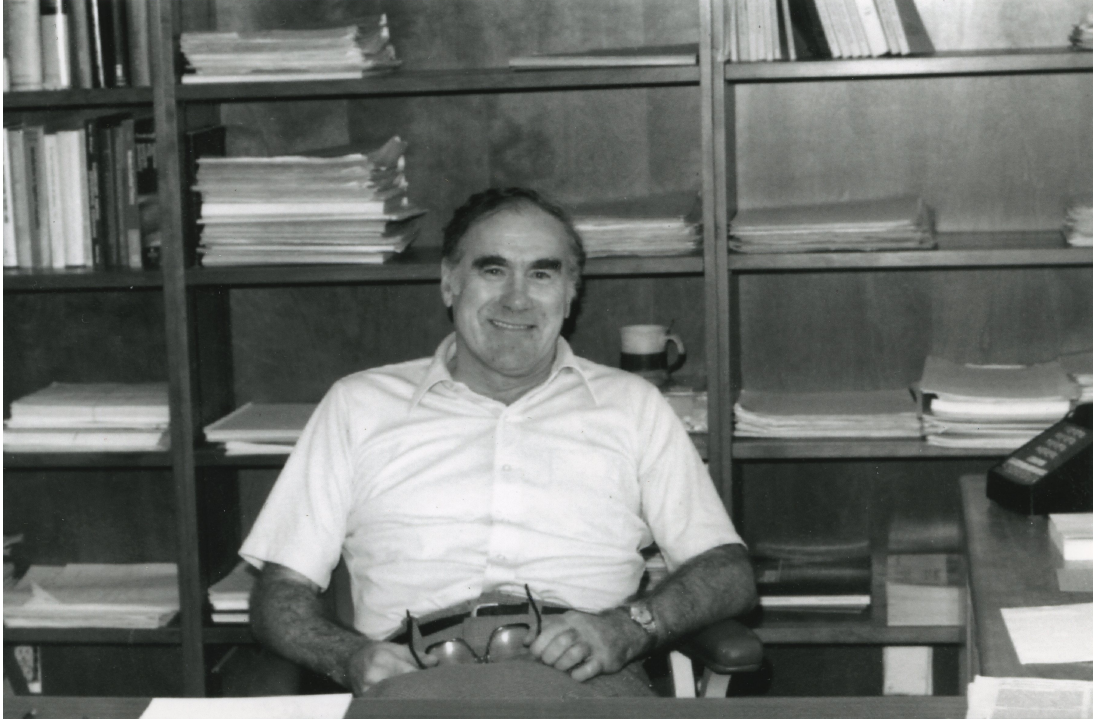


Figure 3: G.B. Whitham in his office in the Firestone Laboratory at the California Institute of Technology. Courtesy of Sheila Shull of the Department of Computing and Mathematical Sciences at the California Institute of Technology.

Whitham's early work on shock waves lead to a deep understanding of the formation and propagation of such waves. This deep understanding lead to an appreciation of the ubiquitous nature of shock waves in physical systems and applications of shock wave theory outside of gas dynamics. In two classic papers [4, 5] Whitham and Lighthill showed how shock waves arise in the evolution of the water flow of a flood and in the dynamics of traffic flow, with both these phenomena governed by systems of nonlinear hyperbolic conservation laws. The first application to flood waves is clearer as the shallow water equations form the same system of hyperbolic equations as the gas equations, with the "ratio of specific heats" being 2. The application of hyperbolic conservation laws and shock waves to the every day experience of traffic flow is not obvious.

At the time, the mid 1950's, there was no quantitative theory for the flow of traffic on roads. As cars are discrete objects, continuum models for their bulk movement would seem to be inappropriate. It is to be credited to Lighthill and Whitham [5] and Richards (1956) that they pursued the idea of modelling the behaviour of individual cars on a single lane highway using conservation laws based on the density of cars, which is valid when this density is not small. The central idea was the realisation that the flow of cars depends in a nonlinear fashion on this density, with the flow being zero at the maximum density, that is stopped bumper to bumper traffic. This central idea was later verified by actual observations of traffic. It should be noted that Lighthill and Whitham modelled the flow, that is flux, of traffic in terms of measurable parameters, such as average car length, drivers' reaction

time and average car velocity. This modelling resulted in the first quantitative understanding of traffic behaviour, enabling quantitative predictions which could be used to design roads, traffic lights and traffic regulations, all these having important consequences for everyday life. One of the main predictions made by Whitham's theory was the propagation of traffic jams as shock waves, with estimates of the speeds of the shock waves. This provided the proof of the nonlinear nature of traffic behaviour. The importance of this work was rapidly recognised in the engineering and design fields. The influence of Whitham's work on traffic flow is detailed in a report by Gerlough and Capelle (1964) and by the fact that nearly all papers in the book a "75 years of the fundamental diagram for traffic flow theory" cite Whitham's work (Transportation Research Board of the National Academies, 2011). It should also be noted that current work on traffic flow theory relies on Whitham's understanding of the subject as a basis for the formulation of traffic control (Transportation Research Board of the National Academies, 2011). Moreover, recent computer models based on particle-type interactions and descriptions are tested by reducing them in the continuum limit to Whitham's equations. Whitham's traffic flow work has become a standard reference with over 3000 ISI citations.

Whitham's work on traffic flow shows how a problem of great complexity can be understood in terms of measurable macroscopic quantities in such a way that the essential physical behaviour is captured. This in turn leads to exploration and prediction, paving the way for a better understanding of a problem of importance to wider society. As traffic flow and traffic jams are a familiar everyday occurrence, the analysis of traffic flow in Whitham's long running course AMa152, "Linear and Nonlinear Waves," generated a lot of interest. So much so, that some Caltech students went to freeways in Los Angeles and measured traffic flow to verify the theory themselves.

While extensive theory existed for the propagation of steady shock waves with uniform fronts (Courant and Friedrichs, 1976), the description of non-steady shocks, especially those with curved fronts, presented a major challenge. To mathematically model such evolving shock waves Whitham developed an approximate theory termed geometrical shock dynamics [7, 8]. Developed in the context of supersonic gas flow, geometrical shock dynamics has, nevertheless, had a subsequent widespread influence on the study of nonlinear diffusion. The main idea of shock dynamics is to relate the deformation of a non-planar shock wave during its propagation to the local velocity of the local shock normal. The dynamics of the shock is then expressed as a functional of the local geometry. This was achieved by Whitham [7, 8] by approximately solving for the propagation of a one dimensional shock in a tube of varying cross section. This in itself is a complicated problem when an approximate solution beyond the small amplitude, Mach number near 1, limit is sought. Whitham obtained a remarkable approximation by a clever analysis which combined the flow at the back of the shock and its influence on the local shock velocity, leading to an analytic approximation for the local shock velocity in terms of the local area of the tube. This demonstrated for the first time the possibility of relating a relevant physical quantity of a propagating front to its actual geometry. The resulting system for the propagating shock front was found to be a hyperbolic system when expressed in terms of the Mach number and the angle of the normal to the shock front. This hyperbolicity has two important consequences. The first is that it shows linear stability for shock waves. The second is the existence of nonlinear instabilities which lead to discontinuities in the shock normal and Mach number along the shock. These were successfully interpreted as Mach triple points. As well as regular reflection consisting of incident and reflected shocks, similar to the reflection of light, shocks have another type of reflection called Mach reflection. In this case, as well as the incident and reflected shocks,

there is a third shock called a Mach stem to which the incident and reflected shocks are joined, the entire configuration having a Y shape with the Mach stem being the bottom of the Y. The point where the three shocks join is called the Mach triple point. Later detailed experiments on shock diffraction were successfully interpreted in terms of geometrical shock dynamics, which to date remains the only analytical theory capable of describing this process. Geometrical shock dynamics was developed before the availability of suitable numerical algorithms to capture shocks and computers powerful enough to implement them. However, even when both of these became available, geometrical shock dynamics has continued to be found useful and to be further developed. The literature is extensive, but examples can be found in the work of Whitham and students [19], Henshaw *et al* (1986) and Schwendeman (1988,1993) and other authors, Best (1993), Besset and Blanc (1994), Cates and Sturtevant (1997), Goodman and MacFadyen (2008) and Anand (2013).

The propagating front closure meant that the dynamical problem could be formulated in geometrical terms. The relevance of this approach for shocks has been described above. It was later realised that the same ideas could be applied to other problems involving propagating fronts (García *et al*, 2001). Such propagating front problems occupy a substantial literature in reaction-diffusion equation theory (Kopell and Howard, 1973). For such reaction-diffusion equations, the problem is to find an equation which relates the propagation of the front normal to a local geometrical property of the front. The analysis assumes, as for shock propagation, that a one dimensional front is stable to local perturbations. A system of local coordinates at the front is then introduced to study the local behaviour of the lateral diffusion. It is shown that the local propagation speed is a linear function of the local curvature. As for shock dynamics, the dynamical process is transformed to a purely geometrical one. In particular, this formulation allows an understanding in simple geometrical terms of threshold behaviour, blocking by geometry and diffraction due to walls for two dimensional diffusion fronts (Kopell and Howard, 1981). This geometrical approach is a very useful first approximation for the interpretation of numerical results of the full system, which due to the inclusion of all effects, can be difficult to interpret. We believe that the geometrical front propagation ideas of Whitham can prove to be very useful in providing a fundamental understanding of these processes.

The shock dynamics just described proved to be very accurate for strong shocks for which the local nature of the flow allows a geometrical description. However, for a large class of problems in weakly supersonic flow a different understanding was needed. Before Whitham's work on the subject it was not possible to assess the effect of geometry on weak shocks and their propagation. Again, Whitham found a key idea which identified the essential features of the problem and provided a means to calculate results in accord with experimental results. This theory is termed nonlinearisation of the acoustic solution [1, 2, 3, 6], also discovered independently by Landau (1945). Whitham realised that the geometry of the weak shock wave was described by the linear acoustic solution. This wave geometry was then used to obtain phases which gave the distortion of the linear rays due to the nonlinear effects which were estimated from the linear geometry. With these ideas, the shock fitting problem became tractable since the wave ahead of and behind the shock were known. It was then possible to find complete solutions for realistic flows long before numerical solutions became available. Indeed, aircraft manufacturers of the time found that Whitham's theory gave good results, but, more importantly, gave valuable insight into complicated flows for which nonlinearity and geometry strongly interacted.

Dispersive Waves

Whitham's greatest legacy in dispersive wave theory is the development of the technique of averaged Lagrangians to study slowly varying linear and nonlinear wavetrains [9, 10, 12, 13, 15]. This theory, termed modulation theory, assumes that the characteristics of a dispersive wavetrain, such as its amplitude, wavenumber, frequency and mean height, are slowly varying functions of space and time. The fast phase oscillation of the wavetrain can then be mathematically decoupled from its slowly varying envelope, resulting in partial differential equations for the slowly varying parameters in slow space and time variables. This averaged Lagrangian technique is related to multiple scales perturbation theory [14]. The key advance of modulation theory is the use of a Lagrangian formulation of the equations, which is averaged by integrating over a wave period in the "fast" phase variable, leaving an averaged Lagrangian in the slow space and time variables. This Lagrangian formulation bypasses many of the difficulties of multiple scales perturbation theory used in its standard "two timing" form (Kevorkian and Cole, 1981). The partial differential equations describing the slowly varying wavetrain parameters are then found as variational equations from the averaged Lagrangian.

One of the key motivations behind the development of modulation theory was the extension of the linear concept of group velocity to nonlinear wave systems [9, 18]. It was then expected that the modulation equations would form a hyperbolic system of partial differential equations, with the characteristic velocities being the nonlinear group velocities [18]. However, it was found for many physical equations governing nonlinear waves that the modulation equations formed an elliptic system, for instance those for surface gravity waves on a fluid [12, 15]. This puzzled Whitham as he expected hyperbolic modulation equations corresponding to real nonlinear group velocities until he heard of experiments by Benjamin in which he had trouble generating the Stokes surface wave on a fluid (water) and suspected that the Stokes wave was unstable (Zakharov and Ostrovsky, 2009, as related by Professor Alan Newell). "The penny then dropped" and Whitham made the connection between stability and the type of the modulation equations (Zakharov and Ostrovsky, 2009, as related by Professor Alan Newell), showing how this resulted from a linear stability analysis [12, 15]. The connection was then made between the classification of the modulation system and the stability of the underlying wavetrain, with hyperbolic modulation equations corresponding to stable wavetrains and elliptic equations corresponding to unstable ones [12, 15]. This key realisation meant that modulation theory was an elegant method in which to determine the stability of nonlinear waves. The Benjamin-Feir side band instability of surface gravity waves (Benjamin, 1967 and Benjamin and Feir, 1967) could then be simply derived [12].

The technique of modulation theory, or averaged Lagrangians, introduced by Whitham made possible the full nonlinear analysis of slow modulations of wavetrains. As stated above, the modulation equations form a system of partial differential equations for the slowly varying wave parameters. When the modulation equations form a hyperbolic system, the modulations travel along the characteristics, with the characteristic velocities the nonlinear counterparts of the linear group velocity. When the modulation equations form an elliptic system, the modulations grow and higher order nonlinearity and dispersion will determine the long term behaviour of the modulated wavetrain. These results show that the instability mechanism is potentially present in all systems, at least those with a Lagrangian formulation, and is caused by the spatio-temporal exchange of wave action among the modulated wave parameters. At the same time, this instability mechanism was found by Zakharov (1967) via a Hamiltonian formulation for weakly nonlinear waves through the Fourier modes of the linear problem. In

the weakly nonlinear regime, the results of Whitham reduce to those of Zakharov (1967). Finally, this instability mechanism was studied experimentally by Benjamin and Feir (1967) and compared with the side band formulation of the Whitham modulation equations [12, 15]. Agreement was found, confirming the physical relevance of the modulation equations. This Whitham-Zakharov-Benjamin-Feir instability became known as modulational instability, which can be present in nonlinear, dispersive physical systems. On learning of Whitham's work on modulation theory, Lighthill (1965) developed it for general weakly nonlinear waves and so derived an explicit criterion for modulational instability. An account of this early work on modulation theory and modulational instability can be found in the review paper of Zakharov and Ostrovsky (2009).

The most far reaching application of modulation theory was the derivation of the modulation equations for the Korteweg-de Vries equation [10, 15], the generic nonlinear wave equation having an exact solution via the inverse scattering method. These modulation equations were derived using a tour-de-force of manipulation of a basic integral and its derivatives. The actual integral was an integral over a period of the underlying cnoidal wavetrain, the integrand itself arising from a cubic polynomial resulting from the differential equation giving the cnoidal wave solution of the Korteweg-de Vries equation. As this loop integral could be expressed in terms of elliptic integrals, this algebraic manipulation was implicitly based on the properties of elliptic integrals and elliptic functions (Whittaker and Watson, 1980). The modulation equations for the Korteweg-de Vries equation were found to be hyperbolic, so that cnoidal waves are modulationally stable. The remarkable finding was that the Riemann invariants for these hyperbolic modulation equations were expressed in terms of the roots of the governing cubic polynomial. This derivation of the modulation equations for a single phase cnoidal wave for the Korteweg-de Vries equation was subsequently extended to multiphase wavetrains by Ablowitz and Benney (1970) using multiple scales perturbation theory (see Dobrokhotov and Maslov, 1982, for a review of this multiple scales theory). Indeed, this multiphase extension directly follows Whitham's ideas on multiphase extensions of his original single phase work [10], displaying the depth of his original insight. The simplicity of the modulation equations for the Korteweg-de Vries equation was unexpected and hinted at a deeper explanation from the inverse scattering solution of the Korteweg-de Vries equation. This deeper explanation was found by Flashka *et al* (1980) using deep results from mathematical analysis, specifically the finite gap theory of Novikov (1974) and Lax (1975). These more powerful techniques also enabled modulation equations for multi-phase wavetrains to be found. Following on from this work, the multi-phase modulation equations for other nonlinear wave equations with inverse scattering solutions have been derived (Pavlov, 1987; Krichever, 1988). In addition, modulation theory could also then be developed for nonlinear wave equations which are a small perturbation of integrable equations (Forest and McLaughlin, 1984; Kamchatnov, 2004).

The other key finding from the modulation equations for the Korteweg-de Vries equation was a result which Whitham was aware of, but did not publish as he was not sure that it was valid from a theory based on a slowly varying wavetrain. The modulation equations for the Korteweg-de Vries equation are hyperbolic and so possess a centred, simple wave solution. Given his previous work on hyperbolic equations, Whitham was well aware of this, but did not think that such a solution would be valid as it corresponded to a jump initial condition, which is not slowly varying. This centred simple wave solution was subsequently derived by Gurevich and Pitaevskii (1974) and was found to correspond to what is termed an undular bore in water wave theory, also termed a dispersive shock and a collisionless shock in plasma physics. When this work appeared, Whitham regretted not publishing it himself. He then collaborated with

Bengt Fornberg and published a classic paper [16] which numerically examined many problems in nonlinear wave theory. One of these was the undular bore solution of the Korteweg-de Vries equation and it was shown that the modulation theory solution for it was in excellent agreement with numerical solutions, once the bore was established. The reason a slowly varying theory gives good solutions for a jump initial condition is that the slowly varying structure of the bore establishes itself rapidly after an initial start-up transient. The undular bore solution of the modulation equations of the Korteweg-de Vries was subsequently justified from the small dispersion limit of the Korteweg-de Vries equation, which corresponds to Bohr-Sommerfeld semi-classical quantum mechanics (Lax and Levermore, 1979). Subsequent to this work on the Korteweg-de Vries equation, modulation theory has diverged into two strands: the derivation and use of simple wave solutions and the more pure mathematical analysis of modulation equations.

The use of modulation theory in physical applications, in particular undular bore solutions, was started by the use of modulation theory for the Korteweg-de Vries equation to analyse resonant flow of a fluid over topography (Grimshaw and Smyth, 1985 and Smyth, 1987). The multiphase modulation equations for the Korteweg-de Vries equation (Flashka *et al*, 1980) are more involved than those for a single phase wavetrain. It was then found that modulation theory could be used to find solutions of initial-boundary value problems for the Korteweg-de Vries equation, for which there is no inverse scattering solution (Marchant and Smyth, 1991, 2002). Once the connection between modulation equations and inverse scattering had been found, the way was then open for deriving the modulation equations for other nonlinear wave equations which possessed an inverse scattering solution, such as the nonlinear Schrödinger equation, the Sine-Gordon equation and the Benjamin-Ono equation. These modulation equations were then used to study a wide variety of physical problems in a wide range of scientific fields, for example nonlinear optics based on the nonlinear Schrödinger equation (Kodama, 1999), Bose-Einstein condensates based again on the nonlinear Schrödinger equation (Kamchatnov, 2004; Hoefer *et al*, 2006), geophysics based on the Sine-Gordon equation (Gershenson *et al*, 2009), oceanography based on the Korteweg-de Vries equation (Smyth and Holloway, 1988) and meteorology based on the Benjamin-Ono equation (Porter and Smyth, 2002). This wide range of scientific applications shows the utility and depth of modulation theory.

The next key advance of modulation theory came from the discovery that to obtain undular bore solutions, the full modulation equations were not needed (El *et al*, 2003; El, 2005). For undular bore solutions, all that was needed was the leading and trailing characteristics of the bore, which could be obtained from the reduced modulation equations for the solitary wave and linear periodic wave solutions, respectively. These reduced modulation equations are available for a large class of nonlinear dispersive wave equations. Since the derivation of tractable modulation equations had previously been essentially limited to equations for which there is an inverse scattering solution, for which it is known that there must be simple modulation equations given by the roots of an underlying polynomial, this opened up the use of modulation theory to a wide variety of non-integrable nonlinear wave equations arising in a wide variety of fields. As for integrable nonlinear wave equations, modulation theory was then applied to a wide range of scientific fields, examples including water waves (El *et al*, 2006; Esler and Pearce, 2011), nonlinear optics (El *et al*, 2007; Crosta *et al*, 2012), Fermi gases (Lowman and Hoefer, 2013a) and geophysics (Lowman and Hoefer, 2013b).

Whitham stated in his book *Linear and Nonlinear Waves* [15] that it would be “extremely valuable to have experimental evidence of this separation, since it is of fundamental impor-

tance is assessing the modulation theory” when referring to the multiple nonlinear group velocities which occur for modulationally stable nonlinear wavetrains. As far as we are aware this experimental verification has not been directly found, but has been indirectly found through comparisons of the predictions of modulation theory, especially the undular bore solution, with experimental and observational results. The first experimental verification of modulation theory was experiments by Tran *et al* (1977) on undular bores in a plasma, governed by the Korteweg-de Vries equation. This work found excellent agreement for the amplitude of the lead soliton of the bore. The lead wave has the largest amplitude and is then relatively less affected by loss and other mechanisms neglected in the Korteweg-de Vries approximation. A similar comparison of the amplitude of the lead wave of a bore generated by coherent light in a nonlinear crystal, governed by the nonlinear Schrödinger equation, was made by Wan *et al* (2006), again giving excellent agreement between modulation theory and experimental results. Good agreement of modulation theory with oceanographic data was obtained by Smyth and Holloway (1988) and with meteorological data by Porter and Smyth (2002). This quantitative comparison of modulation theory and experimental and observational results confirms the soundness of modulation theory and the importance of Whitham’s original insights.

At the same time as modulation theory was being developed and used in applied mathematics, it was also taking a parallel path in pure mathematics. This was due to the connection found by Flashka *et al* (1980) between modulation equations and loop integrals around branch cuts in the complex plane. In pure mathematics, Whitham modulation equations are then associated with the moduli dynamics of Riemann surfaces. This connection then branched out into work on topological field theories, Frobenius manifolds, renormalization groups, coupling constants, and Seiberg-Witten theory, Seiberg-Witten equations, singularity theory, isomonodromy deformations, quantum cohomology and theory, Gromov-Witten invariants and Witten-Dijkgraaf-Verlinde equations (Encyclopaedia of Mathematics, 2002). The connection between the stability of a periodic wavetrain and the elliptic or hyperbolic nature of the modulation equations for it has been rigorously proved (Barker *et al*, 2010; Johnson *et al*, 2010a,b). A few of the more important contributions of Whitham modulation theory to pure mathematics can be listed. Whitham modulation theory, or, more precisely, the structure of the modulation equations for the Korteweg-de Vries equation, was crucial in the construction of a general theory of integrable systems of hydrodynamic type (Tsarev, 1985; Dubrovin and Novikov, 1989). There has also been rigorous work on the Hamiltonian formalism of the Whitham modulation equations (Maltsev, 2010, 2013), following on from the basic theory developed by Whitham [15]. It has even found application in topological field theories (Dubrovin, 1992; Krichever, 1994).

An important discovery for systems of nonlinear diffusion equations was the existence of spiral wave solutions (Cohen, Neu and Rosales, 1978). These waves rotate around a centre and have spiral arms which are related to a phase singularity. There are families of such solutions which are dependent on the amplitude and a wavenumber which is characteristic of the dominant instability which produces these waves. In this context, a modulation theory based on Whitham modulation theory was developed (Hagan, 1982). This formulation clarified the basic mechanisms involved in the evolution of spiral waves under slowly varying conditions. Modulation theory produces a criterion which enables an understanding of the local shape of target patterns. In detail one of the consequences of modulation theory is the formation of solutions with multivalued wavenumber. This is, obviously, not physical, but leads to the conclusion that two spiral waves form. It was later demonstrated that this conclusion is

correct (Kopell and Howard, 1981) and a shock structure in wavenumber gives the transition between the two modulation solutions.

It is seen that the ideas put forward by Whitham in one context have had far reaching consequences in other areas, providing still another example of the insight and ability of Whitham to grasp the essential parts of complicated problems. The need to identify physical mechanisms which could be represented in terms of mathematical ideas, which then could be used to make predictions was always present in Whitham's work. At this stage, we recall several instances which had important impacts on our professional development.

As a first example, comes the realisation that nonlinearity and dispersion in waves could be treated, to a certain extent, with independent descriptions, provided that appropriate conservation equations are preserved in the process. This thought process lead to Whitham's remarkable model for water wave peaking and breaking [12, 15, 16]. This model combines full linear dispersion with a Korteweg-de Vries nonlinearity and gives a remarkable description of the role of deep water wave dispersion in relation to water wave peaking and breaking. This work and the results of this type of modelling have stimulated mathematical work on the Hamiltonian recursive approximation to the water wave equations in terms of Boussinesq-Whitham-type equations (Aceves-Sanchez, Minzoni and Panayotaros, 2013).

The ideas of Whitham, due to their depth and novelty, have been found to be valuable in many fields, and we do not yet know how their ramifications will impact our thinking in the future. A current example of their influence is the study of strongly nonlinear mechanical systems using modulation theory ideas. The need to find solutions of the Whitham equations is stimulating new developments in strongly nonlinear asymptotics.

Teaching and mentoring of junior colleagues

Those of us who had the privilege of attending classes given by G.B. Whitham keep a very vivid memory of what we learnt by listening to the lectures and later doing the carefully selected problems which were given. In class, the ideas were introduced by simple versions of complicated problems for which the main points were presented in a broad picture accompanied by the details which never obscured the main stream. Whitham used to say many times in class that "you have to keep thy thinking straight." Later, one realised that what seemed simple due to Whitham's explanation was the outcome of a great deal of work. The emphasis in class was always on the ideas which made the solution possible. Very often, this was highlighted by explaining the results from different perspectives. In the end, one obtained a first hand exposure as to when and to what extent a scientific problem had been solved. This was one of the most valuable teachings one could receive when aiming to pursue studies in applied mathematics.

In his courses Whitham's presentation was very original, which allowed the coverage of a great deal of material which was necessary to obtain an understanding of the subject matter, accompanied by a working knowledge of the subject. For instance, the course in Stochastic Processes was a wonderful example of this, during which many of the important results for a range of different processes were explained using a combination of differential equations and asymptotic analysis. This combination made it possible for students, after only one year of class, to study works on Brownian motion, filtering of signals, mean exit times, The results were no longer perceived as specialised, but rather as an integral part of the process of generating a mathematical way of predicting natural behaviour.

Whitham's course on Calculus of Variations made students aware of the possibility of

thinking in a consistent way about problems with many degrees of freedom by taking advantage of their variational formulations, with emphasis on the different ways to approximate solutions via the variational setting. This course linked with and was an outshoot of Whitham's research on modulation theory. The correct formulation of boundary conditions, especially for free boundaries, and their approximation proved to be accessible to the class. The ideas of constrained problems and optimal control were made accessible via a number of carefully chosen non-trivial examples which illustrated the convenience of thinking via variational formulations. The students became familiar with the material and gained confidence in a powerful idea for explaining and analysing complicated problems.

Whitham was dedicated to the education and development of his Ph.D. students as well. His wide-ranging knowledge meant that he could suggest problems at the forefront of developments. He was patient with his students, particularly when they were starting their research and knew little, sharing his insight and guiding them to fully fledged researchers in their own right. Whitham had fifteen Ph.D. students in total, many of whom went on to establish academic careers and produce research that has led to new developments in wave theory and fluid mechanics. On the other hand, Whitham had little time for self-promoters who used bluff rather than scientific excellence to advance. A good example of this is his interaction with a beginning post-doctoral fellow. Whitham invited this young scientist into his office to talk about his research. This young scientist then proceeded to tell Whitham that the research he did was out of date and that he should change this post-doctoral fellow's research field. This young scientist did not remain much longer in the Department of Applied Mathematics.

Finally, we remember some of the experience we gained from the advice of and conversations with Whitham while working on problems of mutual interest after we graduated and began our professional lives. The choice of problem was always a very important part of the process. Sometimes Whitham would suggest a broad area of problems which had not been explored. The advice was invariably to concentrate on something for which it was not clear how to proceed. Many times, Whitham advised to look for the nonlinear counterparts of relevant linear effects. On other occasions, he would listen carefully to the problems one found while reading the literature and then point out alternative solutions to these problems which he had derived previously and which he kept in his office drawers. Or you would be pleased to find the solution for a new problem, but on discussing it with Whitham find that he had already done it, with him again pulling out a folder with a neat, carefully detailed solution which, indeed, went far beyond what you had done. Whitham was very selective on what to publish and what was worth publishing. Unfortunately, the wheel has turned and these are not considered virtues now. Whitham had an impressive sense of proportion to put results in their proper context. He was always concerned about the correctness and relevance of the results for understanding the problem at hand. Usually, he pointed out an idea which simplified matters considerably and was never satisfied until a full understanding was obtained.

Whitham's sense of humour while appraising an incomplete or unclear argument was expressed in his phrase "this is clutching at straws." Such words conveyed the urgency of clarifying the matter. The advice which Whitham gave about the possibilities of new ideas which became fashionable at some point of time was always based on a very detached perspective on the actual possibilities of these new ideas in regards to the solution or clarification of actual, physical problems. This perspective was most valuable for junior people, preventing them from becoming stagnant. Whitham had a sly sense of humour which came through

during his lectures. An example of this humour is a story related by Dr. Pat Hagan, a student of Professor Cohen. Whitham was lecturing the introductory graduate applied mathematics course AMa101 “Methods of Applied Mathematics.” During one lecture he presented an example of integrating in the complex plane, “a problem that made grown men weep” as Pat stated. With a twinkle in his eye, he put down the chalk, turned to the audience and said “It’s not so bad if you keep a cool head” and proceeded to pick his way through the problem. This anecdote also displays Whitham’s ability to make extremely difficult material flow and seem clear, that is, until you actually tried to tackle these problems yourself. Whitham always saw teaching and research as two sides of the process of understanding which complemented each other. By example he showed how the process of discovery could be shared with the younger generation and how by this, new questions which generate new ideas arise.

Final Thoughts from Family

Gerald Whitham, our father, moved in 2006 from Southern California to Portland, Oregon, with our mother Nancy Whitham, to be closer to family as his health failed. Gerald and Nancy were married for 58 years, until Nancy’s death in 2009. Nancy’s undergraduate degree in mathematics made her the only immediate family member to fully understand the work that he did, including proof-reading his book *Linear and Nonlinear Waves*. Gerald is survived by his daughters Ruth and Susan, his son Michael, and four grandsons. We miss his sense of humor, his stubborn independence, and the practical life advice that he dispensed to all of us. We appreciate the efforts that Noel Smyth and Tim Minzoni have put into writing our father’s memoir, as the mentoring of graduate students appears to us to have been one of the most satisfying aspects of his long career.

Acknowledgments

The authors wish to thank Gennady El and Luke Sciberras for commenting on sections of this memoir. They also thank Ruth, Michael and Susan Whitham for providing photographs of and personal details on G.B. Whitham and extensive comments on drafts of this memoir. The authors also acknowledge the use of the extensive notes on his life and career which G.B. Whitham wrote before his passing.

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